Math 70 12.6-2nd Log Equations Lesson 42

Objectives:

1) Solve log equations

- when argument contains variable

$$\log_b x = a$$

- when base is the variable

$$\log_x c = a$$

- when log value is the variable

$$\log_b a = x$$

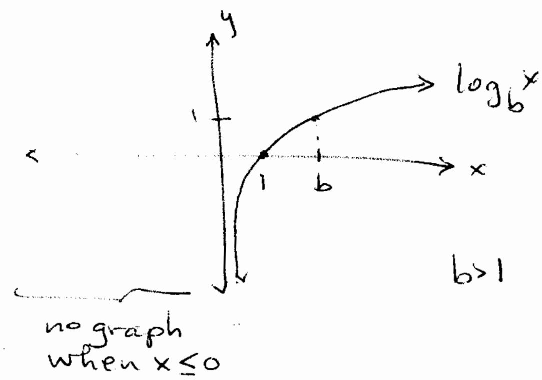
2) Recognize and reject extraneous solutions to log equations

Remember: For any base $b>0$, $b\neq 0$

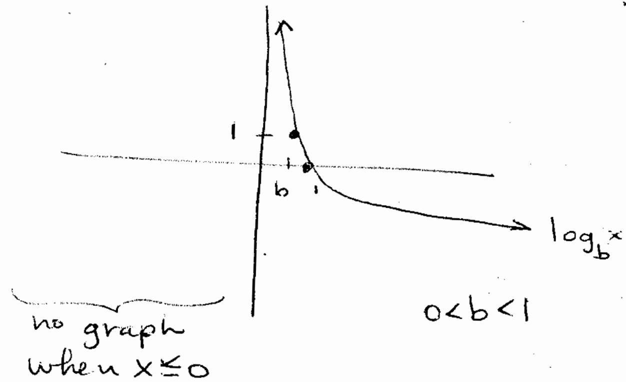
① $\log_b(0)$ = undefined

② $\log_b(-1)$ = undefined

③ $\log_b$(any negative #) = undefined



④ The inverse of the log is ...
the exponential of the same base.



⑤ The inverse of the exponential is ...
the log of the same base.

When solving log and exp eq'ns:

step 1: Isolate the log or exponential. or use log properties to get one log

step 2: Use the inverse function exp → log log → exp

step 3: if log eq'n, check for extraneous.

Solve.

⑥ $\log_4(x-2)=3$

step 1: log is already isolated.

step 2: inverse of log is exponential ⇒ write exponential eq'n.

$$4^3 = x-2$$

$$64 = x-2$$

$$\boxed{66 = x}$$

step 3: check for extraneous

$\log_4(66-2)$ = log (positive #) ✓

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Solve.

(7) $\log_2 x + \log_2 (x-1) = 1$

step 1: we have two logs – use log properties to combine

$$\log_2 x(x-1) = 1$$
$$\log_2 (x^2 - x) = 1$$

step 2: We have a log, use its inverse ⇒ write exponential equation

$$2^1 = x^2 - x$$
$$2 = x^2 - x$$

quadratic equation – set $= 0$, solve by factor, QF, CTS

$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$x = 2,\ x = -1$$

step 3: check for extraneous

$x = 2$: $\log_2(2)$ is defined
$\log_2(2-1) = \log_2(1)$ is defined ✓

$x = -1$ $\log_2(-1)$ is not defined
$x = -1$ is extraneous.

$$\boxed{x = 2}$$

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Solve.

8) $\log_2 x + \log_2(x-2) = 1$

step 1: we have two logs - use log properties to combine

$\log_2 x(x-2) = 1$ $\leftarrow \log_b x + \log_b y = \log_b x \cdot y$

$\log_2(x^2-2x) = 1$

step 2: we have a log, use its inverse $\Rightarrow$ write exponential eqn.

$2^1 = x^2 - 2x$

$2 = x^2 - 2x$

$0 = x^2 - 2x - 2$ doesn't factor ☹

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{4+8}}{2}$

$x = \dfrac{2 \pm \sqrt{12}}{2}$

$x = \dfrac{2}{2} \pm \dfrac{2}{2}\sqrt{3}$

$x = 1 \pm \sqrt{3}$

$x^2 - 2x = 2$

$\# = \left(\frac{-2}{2}\right) = -1$

$\#^2 = (-1)^2 = 1$

$x^2 - 2x + 1 = 2 + 1$

$(x-1)^2 = 3$

$x - 1 = \pm\sqrt{3}$

$x = \pm\sqrt{3}$

step 3: check for extraneous in logs of original equation

$x = 1 + \sqrt{3} \approx 2.7321$

$x = 1 - \sqrt{3} \approx -.7321$

$\log_2(1+\sqrt{3}) = \log_2(+)$ ✓

$\log_2(1+\sqrt{3}-1) = \log(\sqrt{3}) = \log(+)$ ✓

$\log_2(1-\sqrt{3}) \approx \log_2(-.7321) = \log(-)$ ✗ $\leftarrow$ $x = 1-\sqrt{3}$ is extraneous

$\boxed{x = 1 + \sqrt{3}}$

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(8) $\log(x+2) - \log x = 2$

Remember $\log(x+2)$ means $\log_{10}(x+2)$.

$\log\left(\frac{x+2}{x}\right) = 2$ — log property $\log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$

$10^2 = \frac{x+2}{x}$ — exponential equation is inverse of log equation

$100 = \frac{x+2}{x}$

$100x = x+2$ — clear fractions

$99x = 2$

$x = \frac{2}{99}$ — isolate x

$\log\left(\frac{2}{99}+2\right) = \log(+)$ ✓ — check for extraneous

$\log\left(\frac{2}{99}\right) = \log(+)$ ✓

$\boxed{x = \frac{2}{99}}$

(9) Approximate solution using GC, nearest hundredth.

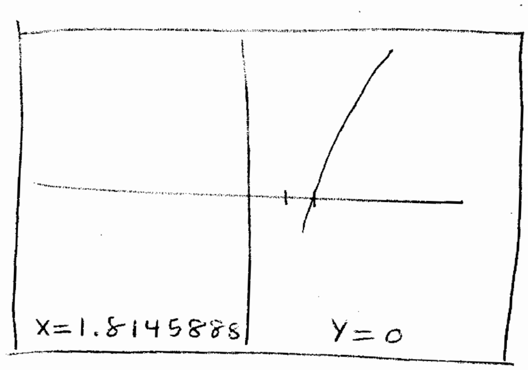
$\ln(1.3x - 2.1) + 3.5x - 5 = 0$

$y_1 = \ln(1.3x - 2.1) + 3.5x - 5$

since eqn = 0

want $y = 0$ → x intercept

⇒ zero



[2nd] [TRACE] = CALC

2. zero

▷ until an x-coord appears, with negative y coordinate, [ENTER]

▷ until y-coord positive, [ENTER] [ENTER]

$x \approx 1.814$ $\boxed{x = 1.81}$

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2 solns

(10) Approximate solution, using GC, to nearest hundredth.

$$2\log(-5.6x + 1.3) = -x - 1$$

$Y_1 = 2\log(-5.6x + 1.3)$

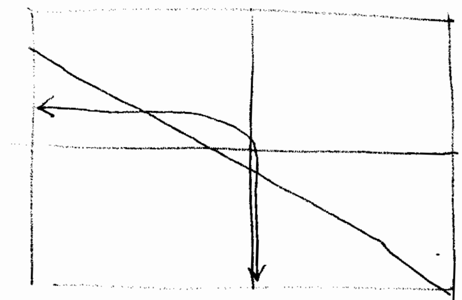
$Y_2 = -x - 1$

remember $\boxed{(-)}$ for x
but $\boxed{-}$ for 1

Since equation $\neq 0$
find intersection.

$\boxed{\text{2nd}}$ $\boxed{\text{TRACE}}$ = CALC
5. intersect

1st curve? $\boxed{\text{enter}}$
2nd curve? $\boxed{\text{enter}}$
Guess? move cursor closer to one of points of intersection



log equation has vertical asymptote

Repeat process to get 2nd solution.

These graphs cross twice, so there are two solutions.

$x \approx -3.681549$
$x \approx .18658985$

$$\boxed{\begin{aligned} x &= -3.68 \\ x &= .19 \end{aligned}}$$

Note: we cannot solve # 10 or # 9 analytically because the variable appears both inside the log and out.

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Extras

Solve

(11) $1 + \log_2 \sqrt{x} = 5$

$\log_2 \sqrt{x} = 5 - 1$

$\log_2 \sqrt{x} = 4$

$2^4 = \sqrt{x}$

$16 = \sqrt{x}$

$(16)^2 = (\sqrt{x})^2$

$\boxed{x = 256}$

check: $1 + \log_2 \sqrt{256}$

$= 1 + \log_2 16$

$= 1 + \frac{\log(16)}{\log(2)}$

$= 5$ ✓

(12) $\ln(4x-5) - \ln(x-2) = \ln(2x+1)$

$\ln\left(\frac{4x-5}{x-2}\right) = \ln(2x+1)$

$\frac{4x-5}{x-2} = 2x+1$ ← exp on e must be the same on LHS as on RHS

$4x-5 = (2x+1)(x-2)$ ← clear frac

$4x-5 = 2x^2 - 4x + x - 2$

$4x-5 = 2x^2 - 3x - 2$

$0 = 2x^2 - 7x + 3$

$0 = (2x-1)(x-3)$

$x = \frac{1}{2} \quad x = 3$

$\ln(\frac{1}{2} - 2) = \ln(-)$ so $x = \frac{1}{2}$ is extraneous

$\left.\begin{aligned} \ln(4 \cdot 3 - 5) &= \ln 7 = \ln(+) \\ \ln(3-2) &= \ln 1 = \ln(+) \\ \ln(2 \cdot 3 + 1) &= \ln 7 = \ln(+) \end{aligned}\right\}$ so $\boxed{x = 3}$ is valid

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(13) $\log(1-2x) = \log 5$

$$1-2x = 5$$
$$-2x = 4$$
$$\boxed{x = -2}$$

check: $\log(1-2\cdot(-2)) = \log(1+4) = \log 5$ ✓

(14) $\frac{1}{2}\ln(3x-1) = \ln 5$

$$\ln\sqrt{3x-1} = \ln 5$$

$$\sqrt{3x-1} = 5$$

$$3x-1 = 25$$ square both sides

$$3x = 26$$

$$\boxed{x = \frac{26}{3}}$$

check:

$\ln\left(3\cdot\frac{26}{3} - 1\right) = \ln(26-1) = \ln 25$ ✓

(15) $\log(275x^2) = 8$

write exponential form, base 10

$$10^8 = 275x^2$$

10^8 is enormous! Don't calculate it

divide both sides by 275 to isolate x^2

$$\frac{10^8}{275} = x^2$$

Square root both sides

$$\boxed{x = \pm\sqrt{\frac{10^8}{275}}} = \boxed{\pm\sqrt{\frac{4000000}{11}}}$$

approx value (if instructions ask to round...)

$x \approx \pm 603.0227$

Is $-\sqrt{\frac{10^8}{275}}$ extraneous? no. In the original, x is squared.

(16) $\log(492x) = 5.728$

Write exponential form, base 10

$$10^{5.728} = 492x$$

divide by 492 to isolate x

$$\boxed{\frac{10^{5.728}}{492} = x}$$ exact

If instructions say to round...

$x \approx 1086.5129$

(17) $\frac{3.01}{\ln x} = \frac{28}{4.31}$

cross-multiply to clear fractions

$$(3.01)(4.31) = 28 \ln x$$

$$12.9731 = 28 \ln x$$

divide by 28 to isolate the logarithm

$$\frac{12.9731}{28} = \ln x$$

rewrite with log on LHS

$$\ln x = \frac{12.9731}{28} = .463325$$

Write in exponential form, base e

$$\boxed{e^{.463325} = x}$$ exact

If instructions say to round...

$x \approx 1.5893$

(18) $\underbrace{\log 692}_{} + \log x = \log 3450$

just a number! subtract from both sides

$$\log x = \log 3450 - \log 692$$

use log property $\log_b a - \log_b c = \log_b \frac{a}{c}$

$$\log x = \log\left(\frac{3450}{692}\right)$$

If two logs have equal bases and are equal to each other then the arguments must be equal!

$$x = \frac{3450}{692}$$

$$\boxed{x = \frac{1725}{346}}$$

Method 2: Move all logs to LHS, combine using log prop

$$\log 692 + \log x - \log 3540 = 0$$

$$\log\left(\frac{692x}{3540}\right) = 0$$

write in exponential form, base 10

$$10^0 = \frac{692x}{3540}$$

simplify exponent

$$1 = \frac{692x}{3540}$$

isolate x

$$3540 = 692x$$

$$\boxed{\frac{3540}{692} = x}$$